Floating Black Holes in Braneworld

Classical BH evaporation conjecture

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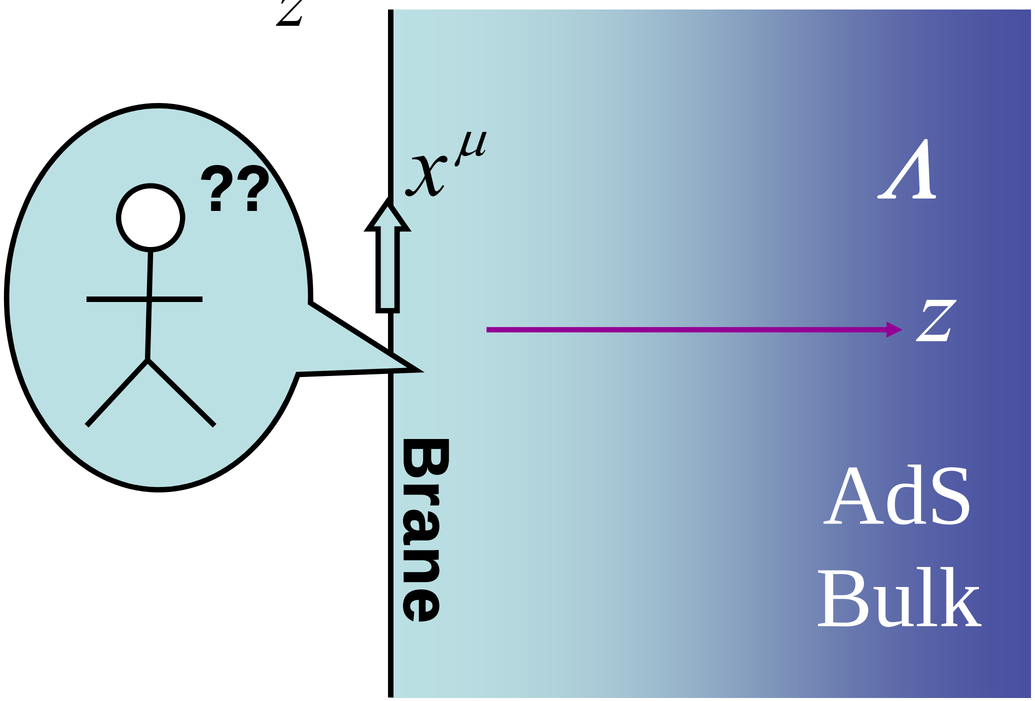
$+\alpha$ (work in collaboration with
N. Tanahashi, K. Kashiyama, A. Flachi**)**

Infinite extra-dimension: Randall-Sundrum II model

Volume of the bulk is finite due to warped geometry although its extension is infinite.

$$ds^2 = \frac{\ell^2}{z^2}\left(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu\right)$$



$z = \ell$

ℓ : AdS curvature radius

$\Lambda = -\frac{6}{\ell^2}$ Negative cosmological constant

$\sigma = \frac{3}{4\pi G_5 \ell}$ Brane tension

- Extension is infinite, but 4-D GR seems to be recovered!

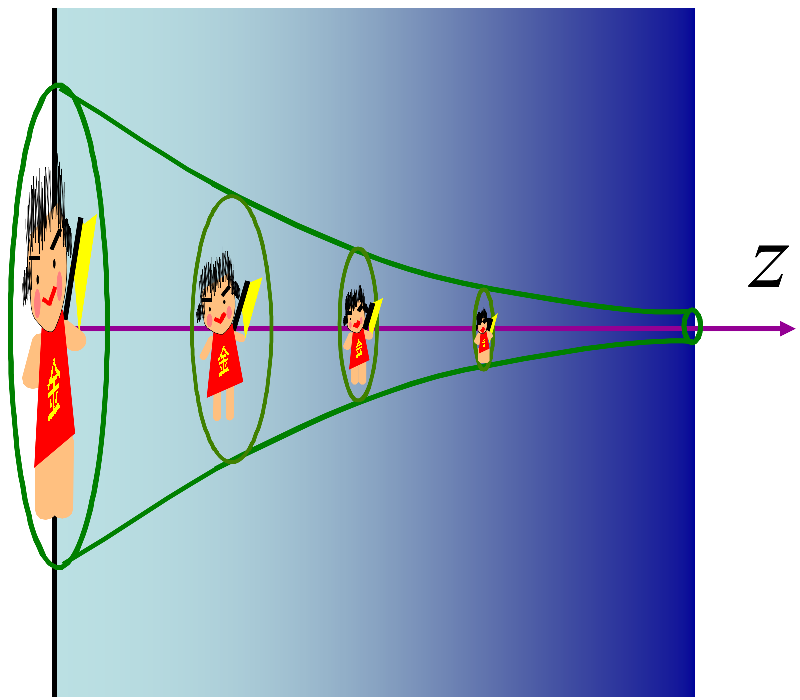
BUT

- No Schwarzshild-like BH solution????

Black string solution

(Chamblin, Hawking, Reall ('00))

$$ds^2 = \frac{\ell^2}{z^2}\left(dz^2 + \bar{g}^{(Sch)}_{\mu\nu} dx^\mu dx^\nu\right)$$



Metric induced on the brane $\bar{g}_{\mu\nu}(x)$ is exactly Schwarzschild solution.

However, this solution is singular.

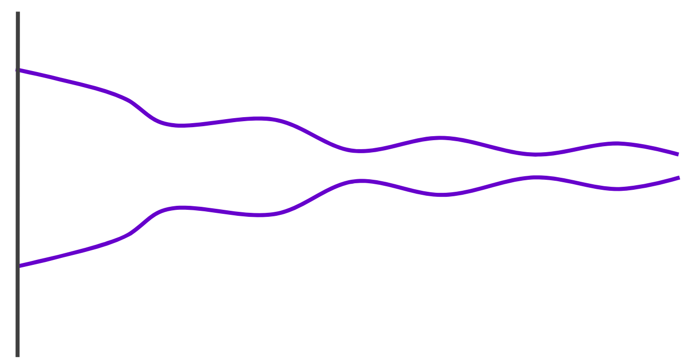
- $C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} \propto z^4$

behavior of zero mode

Moreover, this solution is unstable.

- *Gregory Laflamme instability*

"length $\gtrsim$ width"



Classical black hole evaporation conjecture

(T.T. ('02), Emparan et al ('02))

4D Einstein+CFT with the lowest order quantum correction

$$\left(\begin{array}{c}\text{number of}\\ \text{field of CFT}\end{array}\right) \approx \frac{\ell^2}{\kappa_4^2}$$

AdS/CFT correspondence

⟺ equivalent

Classical 5D dynamics in RS II model

4D BH with CFT ⟺ equivalent ⟺ 5D BH on brane

Hawking radiation in 4D Einstein+CFT picture ⟺ equivalent ⟺ Classical evaporation of 5D BH

Time scale of BH evaporation

$$\tau = \left(\frac{M}{M_{Solar}}\right)^3 \left(\frac{1\text{mm}}{\ell}\right)^2 \times 120\text{year}$$

$$\frac{\dot{M}}{M} \approx \left(\begin{array}{c}\text{Number of}\\ \text{species}\end{array}\right) \times \frac{1}{G_N^2 M^3} \approx \frac{\ell^2}{(G_N M)^3}$$

$10 \pm 5 M_{\odot}$ BH+K-type star X-ray binary A0620-00

ℓ < 0.132mm ($10 M_{\odot}$ BH is assumed)

(Johansen, Psaltis, McClintock arXiv:0803.1835)

(Johansen arXiv:0812.0809)

Numerical brane BH

Kudoh, Nakamura & T.T. (‘03)
Kudoh (’04)

- Static and spherical symmetric configuration

$$ds^2 = \frac{\ell^2}{z^2}\left(-T^2 dt^2 + e^{2R}(dr^2 + dz^2) + r^2 e^{2C} d\Omega^2\right)$$

T, R and C are functions of z and r.

It becomes more and more difficult to construct brane BH solutions numerically for larger BHs.

Small BH case ($\kappa^{-1} < \ell$) is beyond the range of validity of the AdS/CFT correspondence.

Numerical error?
or
Physical ?

Yoshino (’09)

Model with detuned brane tension

Karch-Randall model

JHEP0105.008(2001)

$$S = -\int d^5x \frac{\sqrt{-g}}{2\kappa_5}\left[R - 2\Lambda\right] - \sigma \int \sqrt{-g^{(4)}}\, d^4x$$

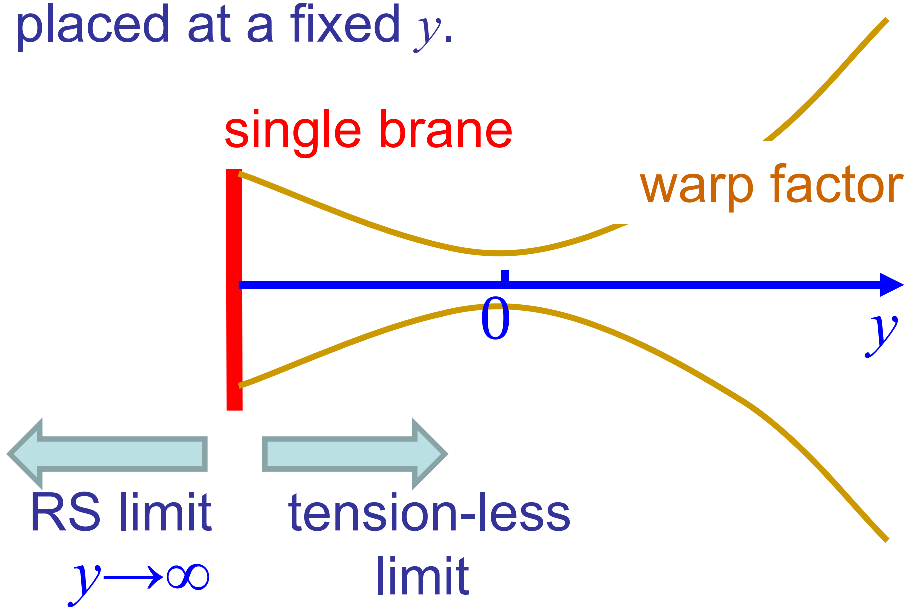
$$\sigma < \sigma_{\rm RS}$$

Effectively four-dimensional negative cosmological constant

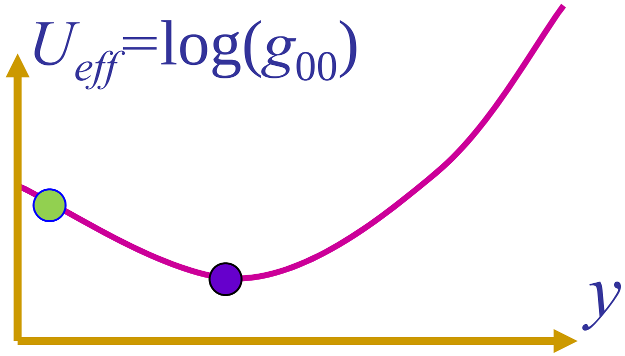
Background configuration:

$$ds^2 = dy^2 + \ell^2 \cosh^2(y/\ell)\, ds^2_{AdS_4}$$

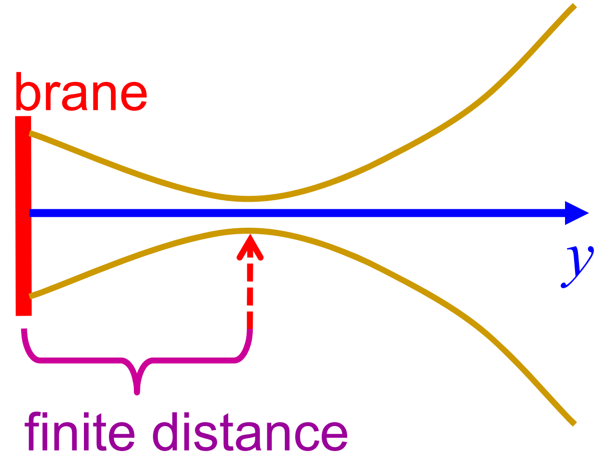
Brane placed at a fixed y.



Effective potential for a test particle (=no self-gravity).



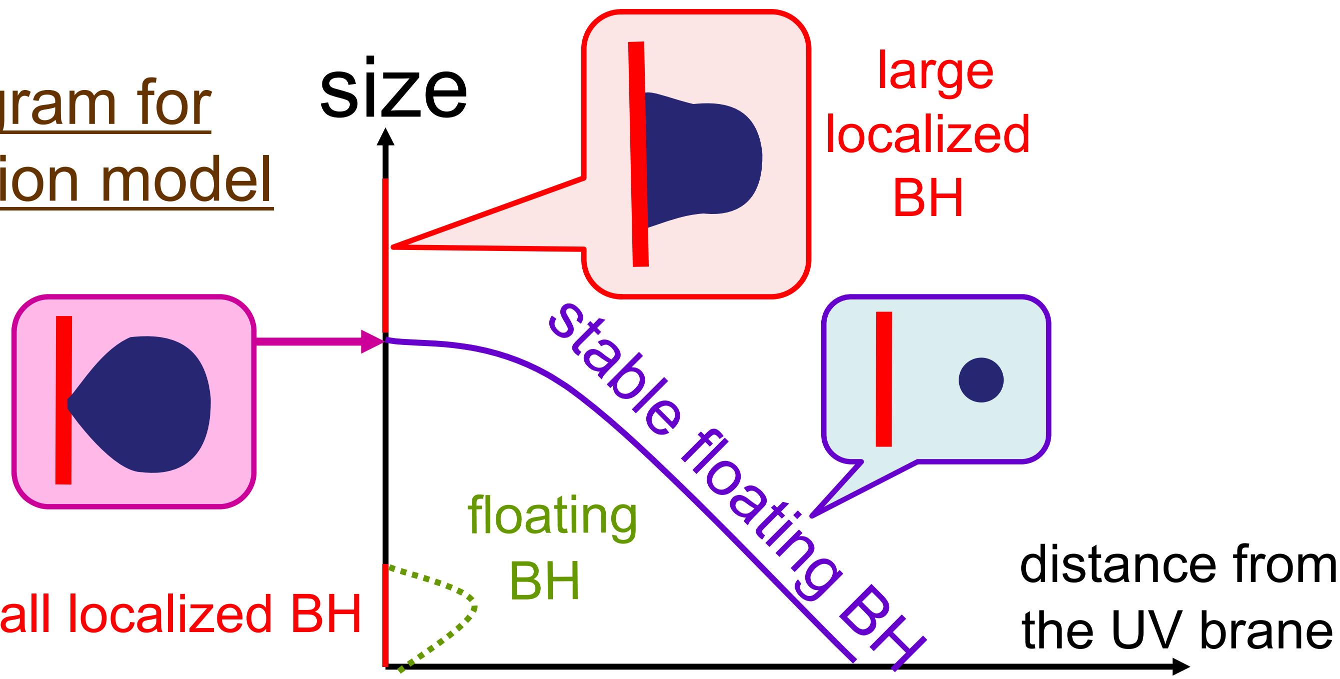
There are stable and unstable floating positions.



➡ Very large BHs cannot float,

➡ necessarily touch the brane.

Phase diagram for detuned tension model



Large localized BHs above the critical size are consistent with AdS/CFT?

Why doesn't static BHs exist in asymptotically flat spacetime?

⇒

Hartle-Hawking (finite temperature) state has regular $T_{\mu\nu}$ on the BH horizon, but its fall-off at large distance is too slow to be compatible with asymptotic flatness.

In AdS, temperature drops at infinity owing to the red-shift factor.

$$T \propto 1/\sqrt{g_{00}} = 1/\sqrt{1-\mu r^{-1} + (r/L)^2}$$

← 4D AdS curvature scale

Quantum state consistent with static BHs will exist if the BH mass is large enough:

$$m_{BH} > m_{pl}^2 (\ell L)^{1/2}.$$

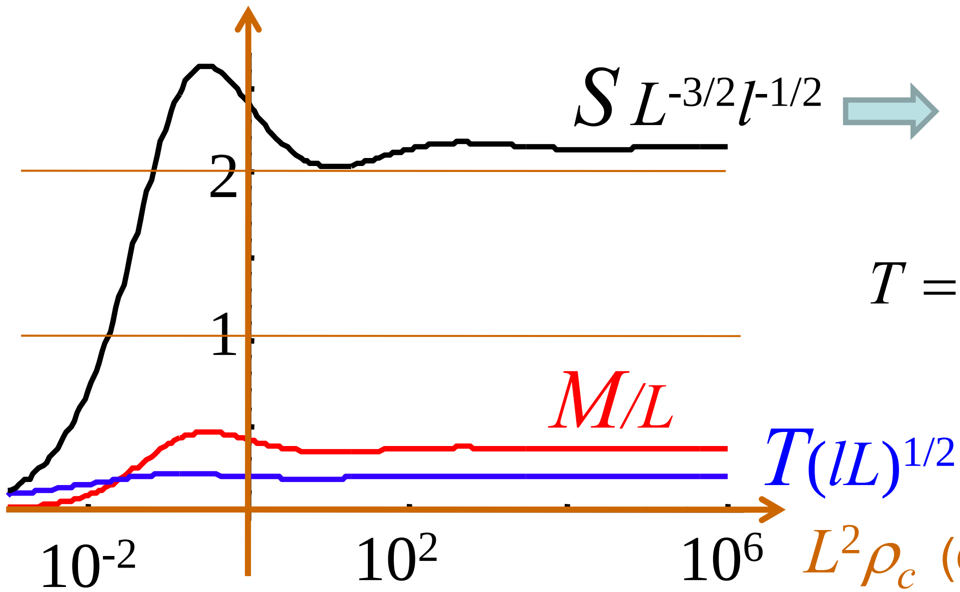
(Hawking & Page '83)

CFT star in 4D GR as counter part of floating BH

4-dimensional static asymptotically AdS star made of thermal CFT ⟺ Floating BH in 5D

The case for radiation fluid has been studied by Page & Phillips (1985)

$$\rho = 3P = aT_{loc}^4 \propto 1/g_{00}^2$$



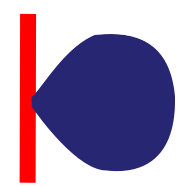
⇒ ~2.15

$$T = \lim_{r\to\infty} T_{loc} \frac{r_{circ}}{L}$$

$L^2\rho_c$ (central density)

Sequence of static solutions does not disappear until the central density diverges.

$\rho \to \infty$ ⇒ $g_{00} \to 0$ ⇒ In 5D picture, BH horizon will be going to touch the brane

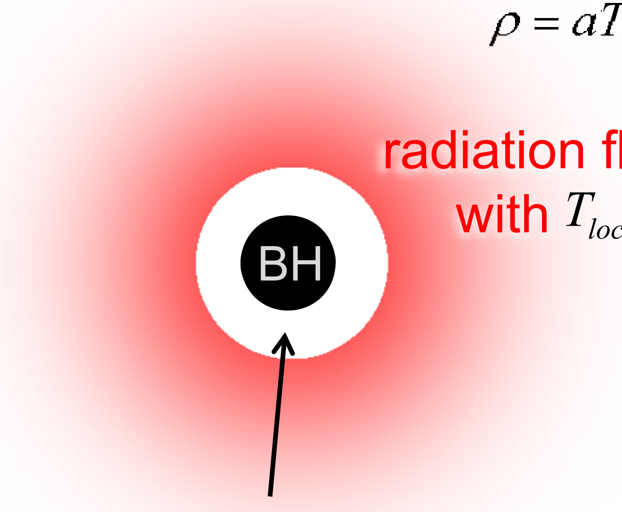


Sequence of sols with a BH in 4D CFT picture

4-dimensional asymptotically AdS space with radiation fluid+BH

Naively, energy density of radiation fluid diverges

$$\rho = aT_{loc}^4 \propto 1/g_{00}^2$$



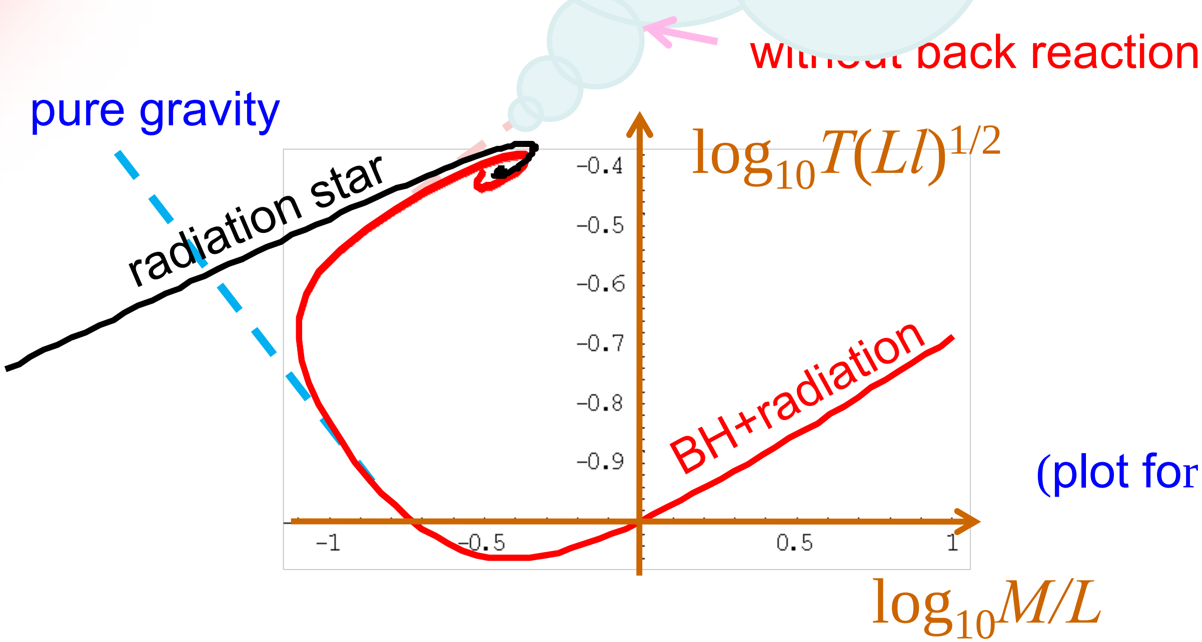
radiation fluid
with $T_{loc} = T_{BH}/\sqrt{-g_{00}}$

empty zone with thickness $\Delta r \sim r_h$

Temperature for the Killing vector ∂_t normalized at infinity,

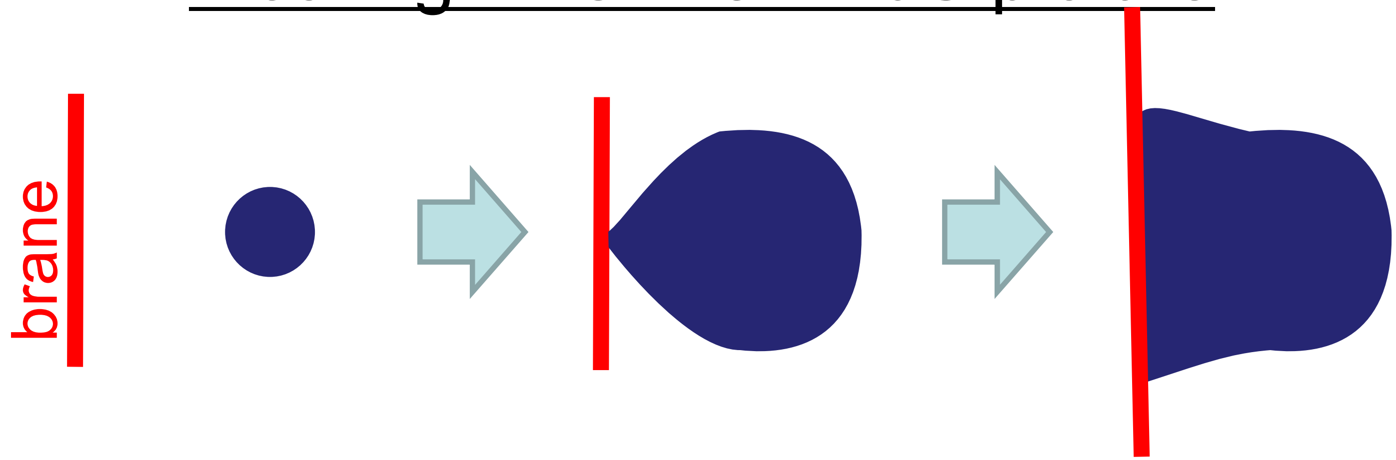
$$T = \lim_{r\to\infty} T_{loc} \frac{r_{circ}}{L}$$

, does not diverge even in the limit r_h –> 0.



(plot for $l=L/40$)

Floating BHs in 5D AdS picture



Numerical construction of static BH solutions is necessary.

However, it seems difficult to resolve two different curvature scales l and L simultaneously. We are interested in the case with $l << L$.

We study time-symmetric initial data just solving

extrinsic curvature of t-const. surface $K_{\mu\nu}=0$.

the Hamiltonian constraint,

$$R_{tt} - \frac{1}{2}Rg_{tt} + \Lambda g_{tt} = 8\pi T_{tt}$$

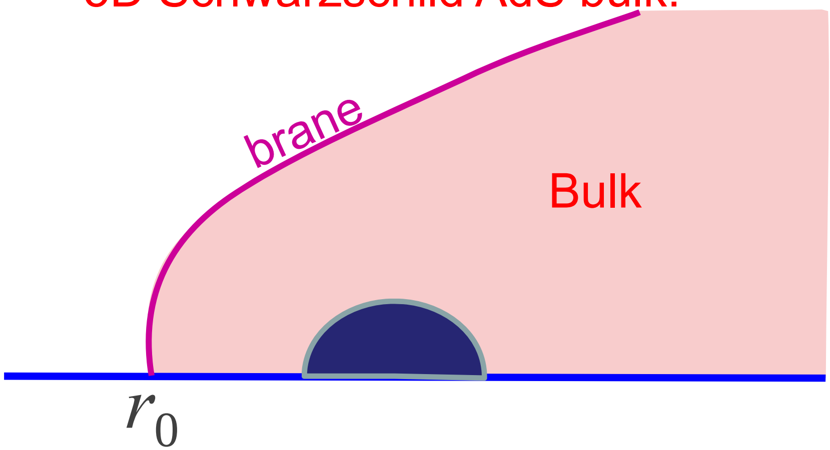
Time-symmetric initial data for floating BHs

work in progress N. Tanahashi & T.T.

We use 5-dimensional Schwarzschild AdS space as a bulk solution.
⇨ Hamiltonian constraint is automatically satisfied in the bulk.

Then, we just need to determine the brane trajectory to satisfy the Hamiltonian constraint across the brane.

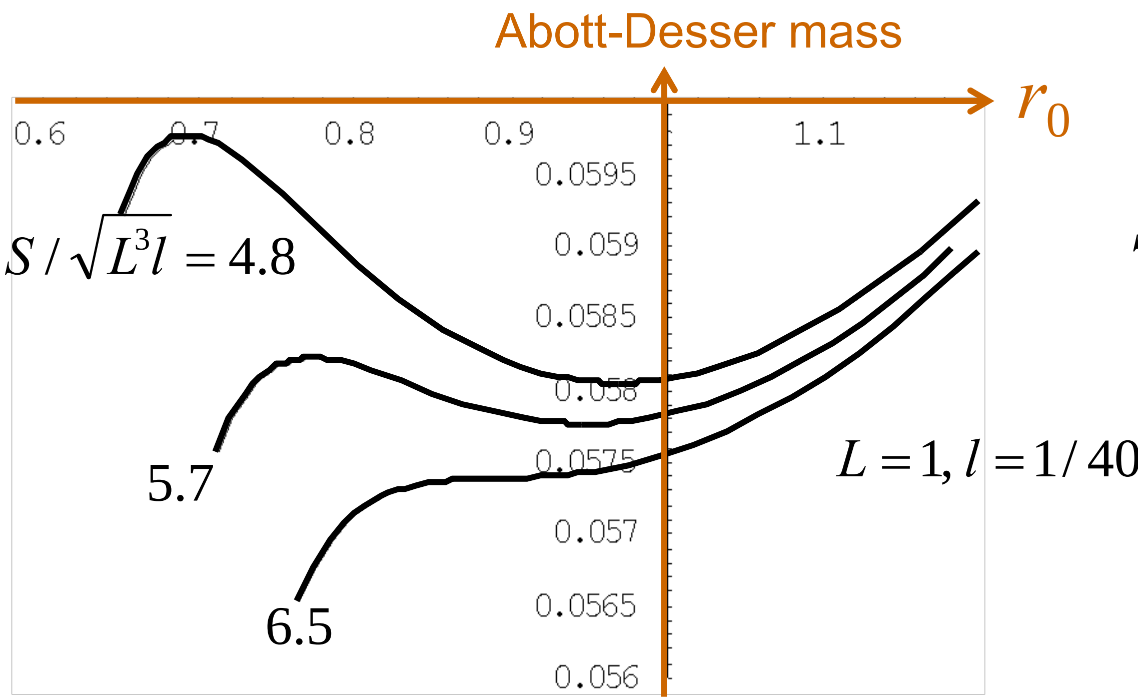
5D Schwarzschild AdS bulk:



$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + r^2 d\Omega^2$$

Brane:=3 surface in 4-dimensional space.
t=constant slice

Hamiltonian constraint on the brane ⇔ $\left\{ \text{Trace of extrinsic curvature of this 3 surface} \right\} = 3\sqrt{\frac{1}{l^2} - \frac{1}{L^2}}$



$$S = \frac{2\times(Area)}{4G_4 l}$$

Critical value where mass minimum (diss)appears is approximately read as

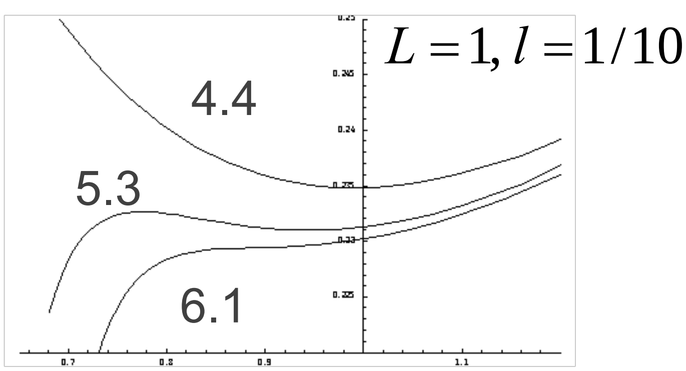
$$S/\sqrt{L^3 l} \approx 6$$

Critical value is close to

$$S_{crit}\sqrt{L^3 l} \approx 3.6$$

expected from the 4dim calculation,

and it is almost independent of *l/L*.



Summary

- AdS/CFT correspondence suggests that there is no static large $(\kappa^{-1} \gg \ell)$ brane BH solution in RS-II brane world.
  - This correspondence has been tested in various cases.
- Small localized BHs were constructed numerically.
  - The sequence of solutions does not seem to terminate suddenly,
  - but bigger BH solutions are hard to obtain.
- We presented a scenario for the phase diagram of black objects including Karch-Randall detuned tension model,

  which is consistent with AdS/CFT correspondence.

  As a result, we predicted new sequences of black objects.

  1) floating stable and unstable BHs

  2) large BHs localized on AdS brane

- Partial support for this scenario was obtained by comparing the 4dim asymptotic AdS isothermal star and the 5dim time-symmetric initial data for floating black holes.